Small Area Estimation: A Novel Approach on Estimation of Mean Squared Prediction Error of Small-Area Predictors

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Outline

- Motivation

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- Motivation
- Model Framework
  - Small Area Predictors
  - Measures of Uncertainty

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- Conclusions

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Motivation

- Mixed models are widely used for analyzing correlated data which cover cross-sectional, spatial, and so on.
- An important case of mixed models is the generalized linear mixed model (GLMM) which has extensively been used in the context of small area estimation (SAE; Rao and Molina, 2015) to cover normal and non-normal responses.
- Normal response: state (small area) estimates of median income of four-person families
  Non-normal response: proportions of persons without health insurance for different minority groups (small areas)

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Motivation

- However, we do not have enough sample to provide direct estimate (e.g., age-sex-race domains); we use the mixed models to borrow strength from other resources to get reliable estimates.
- A major topic in SAE is estimation of mean squared prediction errors (MSPEs) for predictors of various characteristics of interest associated with the small areas.
- A "gold standard" for the MSPE estimation is to produce a second-order unbiased MSPE estimator, that is, the order of bias of the MSPE estimator is $o(m^{-1})$, where m is the number of small areas from which data are collected.

Motivation

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- Currently, there are two approaches for producing a second-order unbiased MSPE estimator.
- Prasad-Rao linearization method (Prasad and Rao, 1990) which uses Taylor series expansion (tedious derivations; complicated expressions; does not apply to non-differentiable operations such as model selection and shrinkage estimation)

Motivation

- Resampling Techniques:
  - Jackknife methods:

    - Jackknife method (JLW: Jiang et al., 2002) does not apply to non-normal random effects nor to a predictor that is obtained post model selection (PMS).
    - Jiang et al. (2018) proposed a Monte-Carlo jackknife method (McJack) which leads to a second-order unbiased MSPE estimator in situations like PMS; however, it is computationally intensive.

  - Double bootstrap methods (Hall and Maiti, 2006):

    - Although double bootstrap (DB) is capable of producing a second-order unbiased MSPE estimator, it is, perhaps, computationally even more intensive than the McJack.

Objective

- Propose a second-order unbiased MSPE estimation of small area predictors:

  • which is a hybrid of the linearization method and resampling method, by combining the best part of each method; it is also less computationally intensive compared to the McJack and DB.

Generalized Linear Mixed Model (GLMM)

- Exponential family probability density or mass function:

$$f(y_i|\theta_i, \phi_i) = \exp[\{y_i\theta_i - a(\theta_i)\}/\phi_i + b(y_i, \phi_i)],$$

$$(i = 1, ..., m),$$

y_i : variable of interest for the $i-$th small area,
θ_i : canonical parameters,
ϕ_i : known scale parameters,
$a(\cdot)$ and $b(\cdot)$: known functions,
m : number of small areas

Generalized Linear Mixed Model (GLMM)

- Natural parameters θ_i :

$$g(\theta_i) = x_i'\beta + v_i,$$

$g(\cdot)$: link function,
$v_i \sim N(0, A)$,
$\psi = (\beta, A)$: model parameters

Mean Squared Prediction Error (MSPE)

- Let $\hat{\theta}$ (suppress i for notation simplicity) be a predictor of θ, that is, a function of the observed data, y:

  • $\hat{\theta}$ can be: EBLUP, or EBP; PMS EBLUP or PMS EBP, to which the standard methods such as Prasad-Rao and JLW do not apply to obtain a second-order unbiased MSPE estimator

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- The MSPE of $\hat{\theta}$ can be expressed as:

$$\mathrm{MSPE} = \mathrm{E}(\hat{\theta} - \theta)^2 = \mathrm{E}\left[\mathrm{E}\{(\hat{\theta} - \theta)^2 | y\}\right] \tag{1}$$

Mean Squared Prediction Error (MSPE)

- From (1), we can write:

$$\begin{aligned} a(y,\psi) &= \mathrm{E}\{(\hat{\theta}-\theta)^2|y\} \\ &= \hat{\theta}^2 - 2\hat{\theta}\mathrm{E}(\theta|y) + \mathrm{E}(\theta^2|y) \\ &= \hat{\theta}^2 - 2\hat{\theta}h_1(y,\psi) + h_2(y,\psi), \end{aligned}$$

where $h_j(y,\psi) = \mathrm{E}(\theta^j|y), j = 1, 2.$

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where $h_j(y,\psi) = \mathrm{E}(\theta^j|y), j = 1, 2.$

- Replace ψ by its consistent estimator $\hat{\psi}$:

$$\mathrm{E}\{a(y,\hat{\psi}) - a(y,\psi)\} = O(m^{-1})$$

Mean Squared Prediction Error (MSPE)

- In other words:

$$d(\psi) = b(\psi) - c(\psi) = O(m^{-1}),$$

where

$$b(\psi) = \mathrm{MSPE} = \mathrm{E}\{a(y,\psi)\},$$
$$c(\psi) = \mathrm{E}\{a(y,\hat{\psi})\}$$

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- One can then show that:

$$d(\hat{\psi}) - d(\psi) \quad = \quad o_p(m^{-1})$$

MSPE Estimation

- Under some regularity conditions:

$$\widehat{\mathrm{MSPE}} = a(y, \hat{\psi}) + b(\hat{\psi}) - c(\hat{\psi}), \qquad (2)$$

in the sense that $\mathrm{E}(\widehat{\mathrm{MSPE}}) = \mathrm{MSPE} + o(m^{-1})$.

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- We can calculate $b(\hat{\psi})$ and $c(\hat{\psi})$ by Monte Carlo methods.

MSPE Estimation

- Let $y_{[k]}$ denote y generated under the kth Monte-Carlo sample, $k = 1, \ldots, K$. Then, we have:

$$b(\psi) - c(\psi) \quad \approx \quad \frac{1}{K} \sum_{k=1}^{K} \left\{ a(y_{[k]}, \psi) - a(y_{[k]}, \hat{\psi}_{[k]}) \right\},$$

where $\hat{\psi}_{[k]}$ denotes $\hat{\psi}$ based on $y_{[k]}$

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where $\hat{\psi}_{[k]}$ denotes $\hat{\psi}$ based on $y_{[k]}$

- A Monte-Carlo assisted second-order unbiased MSPE estimator (called Sumca: simple, unified, Monte-Carlo assisted) is given by

$$\begin{aligned}\widehat{\mathrm{MSPE}}_K &= a(y, \hat{\psi}) + d_K(\hat{\psi}) \\ &= a(y, \hat{\psi}) + \frac{1}{K}\sum_{k=1}^{K}\left\{a(y_{[k]}, \hat{\psi}) - a(y_{[k]}, \hat{\psi}_{[k]})\right\} \quad (3)\end{aligned}$$

MSPE Estimation

- Remark: A special form of the leading term, $a(y, \hat{\psi})$, deserves attention. We can write

$$a(y, \psi) = \{\hat{\theta} - E(\theta|y)\}^2 + var(\theta|y)$$

• the first term on the right side of the above equation vanishes when ψ is replaced by $\hat{\psi}$.
• under the general linear mixed model, we then have $a(y, \hat{\psi}) = var(\theta|y) = V(\gamma)$, γ is a vector of dispersion parameters, which shows stability of the leading term of the proposed MSPE estimator.

MSPE Estimation

- Additional remarks:

  • The leading term, $a(y, \hat{\psi})$, in the proposed MSPE estimator is guaranteed positive.

  • The Sumca estimator is computationally much less intensive than McJack; McJack requires m^2/K goes to 0, while the Sumca does not have such a restriction (it is recommended that $K = m$ in standard situations). For example, if $m = 100$, the computational cost for Sumca is about 0.001% to 0.01% of that of McJack.

Fay-Herriot Model

- Fay-Herriot (FH) model (1979):

$$\begin{aligned} y_i &= x_i'\beta + v_i + e_i \\ &= \theta_i + e_i, \quad i = 1, \ldots, m, \end{aligned}$$

where $v_i \sim N(0, A), e_i \sim N(0, D_i)$

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where $v_i \sim N(0, A)$, $e_i \sim N(0, D_i)$

- MSPE estimation of $\hat{\theta}_i$ proposed by Prasad-Rao (1990):

$$\widehat{\mathrm{MSPE}}_{i,\mathrm{PR}} = \frac{\hat{A}D_i}{\hat{A}+D_i} + \left(\frac{D_i}{\hat{A}+D_i}\right)^2 x_i' \left(\sum_{j=1}^{m} \frac{x_j x_j'}{\hat{A}+D_j}\right)^{-1} x_i + \frac{4D_i^2}{(\hat{A}+D_i)^3 m^2} \sum_{j=1}^{m} (\hat{A}+D_j)^2$$

Fay-Herriot Model

- Sumca estimator of $\hat{\theta}_i$:

$$\widehat{\text{MSPE}}_i = \frac{\hat{A}D_i}{\hat{A}+D_i} + \frac{1}{K}\sum_{k=1}^{K}\left\{a_i(y_{[k]},\hat{\psi}) - a_i(y_{[k]},\hat{\psi}_{[k]})\right\},$$

where

$$a_i(y,\psi) = \frac{AD_i}{A+D_i} + \left(\hat{\theta}_i - \frac{A}{A+D_i}y_i - \frac{D_i}{A+D_i}x_i'\beta\right)^2$$

Area-level Model with Model Selection

- To test $H_0 : A = 0$ in the FH model (DHM: Datta, Hall, Mandal, 2011):

$$\hat{\theta}_i = \begin{cases} \frac{\hat{A}}{\hat{A}+D_i} y_i + \frac{D_i}{\hat{A}+D_i} x_i' \hat{\beta}, & \text{if } T > \chi^2_{m-p}(1-\alpha) \\ x_i' \tilde{\beta}, & \text{if } T \leq \chi^2_{m-p}(1-\alpha) \end{cases} \quad (4)$$

where $T = \sum_{i=1}^{m} D_i^{-1}(y_i - x_i'\hat{\beta})^2, \tilde{\beta} = (X'D^{-1}X)^{-1}X'D^{-1}y$

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- MSPE estimation proposed by DHM:

$$\widehat{\mathrm{MSPE}}_{i,\mathrm{DHM}} = \begin{cases} \widehat{\mathrm{MSPE}}_{i,\mathrm{PR}} & \text{if } T > \chi^2_{m-p}(1-\alpha) \\ x_i' (X' D^{-1} X)^{-1} x_i & \text{if } T \leq \chi^2_{m-p}(1-\alpha) \end{cases}$$

Area-level Model with Model Selection

- Sumca applies to any kind of predictor including DHM (4):

$$\widehat{\mathrm{MSPE}}_i = \frac{\hat{A}D_i}{\hat{A}+D_i} + \left(\hat{\theta}_i - \frac{\hat{A}}{\hat{A}+D_i}y_i - \frac{D_i}{\hat{A}+D_i}x_i'\hat{\beta}\right)^2 + \frac{1}{K}\sum_{k=1}^{K}\left\{a_i(y_{[k]},\hat{\psi}) - a_i(y_{[k]},\hat{\psi}_{[k]})\right\},$$

where

$$a_i(y_{[k]},\hat{\psi}) - a_i(y_{[k]},\hat{\psi}_{[k]})$$

$$= D_i\left(\frac{\hat{A}}{\hat{A}+D_i} - \frac{\hat{A}_{[k]}}{\hat{A}_{[k]}+D_i}\right) + \left(\hat{\theta}_i - \frac{\hat{A}}{\hat{A}+D_i}y_{[k],i} - \frac{D_i}{\hat{A}+D_i}x_i'\hat{\beta}\right)^2 - \left(\hat{\theta}_i - \frac{\hat{A}_{[k]}}{\hat{A}_{[k]}+D_i}y_{[k],i} - \frac{D_i}{\hat{A}_{[k]}+D_i}x_i'\hat{\beta}_{[k]}\right)^2$$

Mixed Logistic Model

- Let

$$\mathrm{P}(y_{ij} = 1|v_i) = p_{ij},\ \mathrm{logit}(p_{ij}) = x_{ij}'\beta + v_i, (i = 1, \ldots, m; j = 1, \ldots, n_i)$$

where $v_i \sim N(0, A)$

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where $v_i \sim N(0, A)$

- Assume $x_{ij} = x_i$, then $\theta_i = g(x_i'\beta + v_i)$ where $g(u) = e^u/(1 + e^u)$; $y_i = \sum_{j=1}^{n_i} y_{ij}$, so

$$f_\beta(y_i|v_i) = \exp\left\{y_i(x_i'\beta + v_i) - n_i \log(1 + e^{x_i'\beta + v_i})\right\},$$

$$h_{i,s}(y, \psi) = \mathrm{E}(\theta_i^s|y) = \frac{\int \{g(x_i'\beta + v_i)\}^s f_\beta(y_i|v_i) f_A(v_i) dv_i}{\int f_\beta(y_i|v_i) f_A(v_i) dv_i}, s = 1, 2$$

Mixed Logistic Model

- MSPE estimation of $\hat{\theta}_i$ proposed by Jiang, Lahiri, Wu (JLW) in 2002:

$$\begin{aligned}\widehat{\text{MSPE}}_{i,\text{JLW}} &= B_i(\hat{\psi}) - \frac{m-1}{m}\sum_{i'=1}^{m}\{B_i(\hat{\psi}_{-i'}) - B_i(\hat{\psi})\} \\ &+ \frac{m-1}{m}\sum_{i'=1}^{m}\{\hat{\theta}_{i,-i'} - \hat{\theta}_i\}^2,\end{aligned}$$

where

$$B_i(\psi) = \sum_{k=1}^{n_i}\binom{n_i}{k}\left[h_{i,2}(k,\psi) - \{h_{i,1}(k,\psi)\}^2\right]\mathrm{E}\{\theta_i^k(1-\theta_i)^{n_i-k}\},$$

which is evaluated via numerical integration.

Mixed Logistic Model

- Sumca estimator of $\hat{\theta}_i$:

$$\widehat{\mathrm{MSPE}}_i \;=\; a_i(y, \hat{\psi}) + \frac{1}{K}\sum_{k=1}^{K}\left\{a_i(y_{[k]}, \hat{\psi}) - a_i(y_{[k]}, \hat{\psi}_{[k]})\right\},$$

where

$$a_i(y, \hat{\psi}) = h_{i,2}(y, \hat{\psi}) - \{h_{i,1}(y, \hat{\psi})\}^2$$

$$= h_{i,2}(y, \hat{\psi}) - \hat{\theta}_i^2$$

Fay-Herriot Model

- Fay-Herriot (FH) model:

$$\begin{aligned} y_i &= \beta_0 + \beta_1 x_i + v_i + e_i \\ &= \theta_i + e_i, \quad i = 1, \ldots, m, \end{aligned}$$

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  where $v_i \sim N(0, A)$, $e_i \sim N(0, D_i)$

- Simulation set-up:
  $m = 2m_1$; $D_i = D_{i1} (1 \leq i \leq m_1)$;$D_i = D_{i2}$ $(m_1 + 1 \leq i \leq m)$;
  $\beta_0 = \beta_1 = 1$; $A = 10$; $D_{i1} \sim U(3.5, 4.5)$; $D_{i2} \sim U(0.5, 1.5)$;
  $x_i \sim U(0, 1)$ (x_i, D_{i1}, and D_{i2} are fixed during simulation);
  $R = 1000$: number of simulations

Fay-Herriot Model

- Empirical MSPE (EMSPE):

$$\text{EMSPE}_i = \frac{1}{R}\sum_{r=1}^{R}\{\hat{\theta}_i^{(r)} - \theta_i^{(r)}\}^2, (i = 1, ..., m; r = 1, ..., R)$$

Fay-Herriot Model

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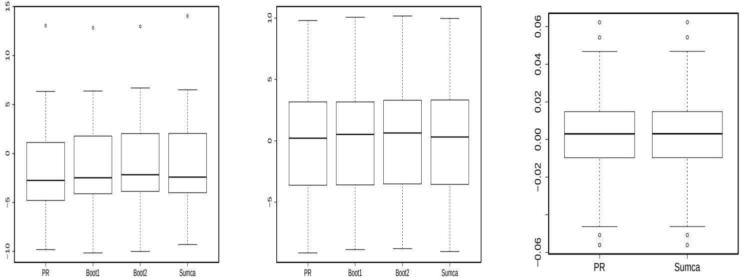
$$\text{EMSPE}_i = \frac{1}{R}\sum_{r=1}^{R}\{\hat{\theta}_i^{(r)} - \theta_i^{(r)}\}^2, (i = 1, ..., m; r = 1, ..., R)$$

- % RB of a MSPE estimator ($\widehat{\text{MSPE}}$):

$$\%\ \text{RB} = 100 \times \left\{ \frac{\text{E}(\widehat{\text{MSPE}}) - \text{EMSPE}}{\text{EMSPE}} \right\}$$

Fay-Herriot Model

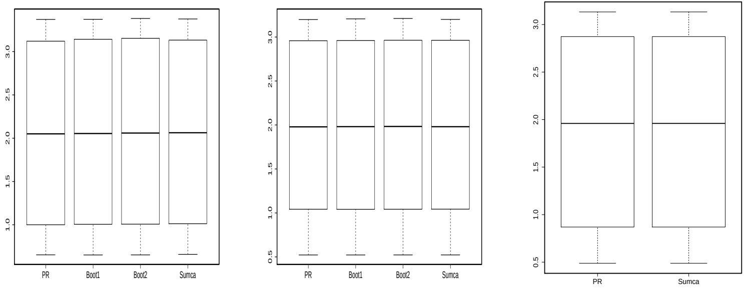
- Figure 1: Boxplots of % RB of MSPE estimates using Prasad-Rao, bootstrap, and Sumca methods: (a) $m = 20$, (b) $m = 50$, and (c) $m = 200$.



(a) (b) (c)

Fay-Herriot Model

- Figure 2: Boxplots of MSPE estimates using Prasad-Rao, bootstrap, and Sumca methods: (a) $m = 20$, (b) $m = 50$, and (c) $m = 200$.



(a) (b) (c)

Area-level Model with Model Selection

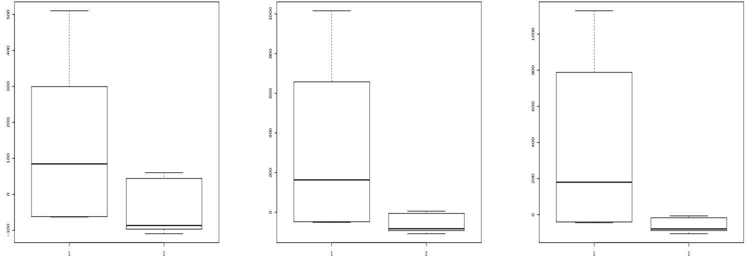
- Simulation set-up:

$$\begin{aligned} y_i &= \beta_{01} + \beta_1 x_i + v_i + e_i, \quad i = 1, ..., m_1, \\ y_i &= \beta_{02} + \beta_1 x_i + v_i + e_i, \quad i = m_1 + 1, ..., m, \end{aligned}$$

$m = 2m_1$; $m = 20$; $D_i = D_{i1} (1 \leq i \leq m_1)$;
$D_i = D_{i2}$ $(m_1 + 1 \leq i \leq m)$; $\beta_{01} = 1; \beta_{02} = 4; \beta_1 = 1$;
$A (= 0, 0.5, 1)$; $D_{i1} \sim U(0.5, 1.5)$; $D_{i2} \sim U(15.5, 16.5)$;
$x_i \sim U(0, 1)$ (x_i, D_{i1}, and D_{i2} are fixed during simulation);
$R = 1000$: number of simulations

Area-level Model with Model Selection

- Figure 3: Boxplots of % RB for Sumca and DHM methods under different values of A at $\alpha = 0.20$. In each plot, Left: DHM; Right: Sumca



$A = 0$ $A = 0.5$ $A = 1$

Health Insurance of Minority Sub-populations Data

- To consider small domain estimation of the proportion of persons without health insurance for different minority groups in the Asian population in the USA

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Health Insurance of Minority Sub-populations Data

- To consider small domain estimation of the proportion of persons without health insurance for different minority groups in the Asian population in the USA
- Data provided by National Health Interview Survey (NHIS) for the year 2000, which report the individual level binary responses on whether a person has health insurance, along with his or her individual level covariates
- The total number of domains (m) is $96 (= 3 \times 2 \times 4 \times 4)$ based on age $\times$ sex $\times$ race $\times$ region.

Health Insurance of Minority Sub-populations Data

- We consider the following model:

$$\mathrm{logit}(p_{ij}) = \beta_0+\beta_1 x_{ij1}+\beta_2 x_{ij2}+\beta_3 x_{ij3}+v_i, (i = 1, ..., 96; j = 1, ..., n_i),$$

where

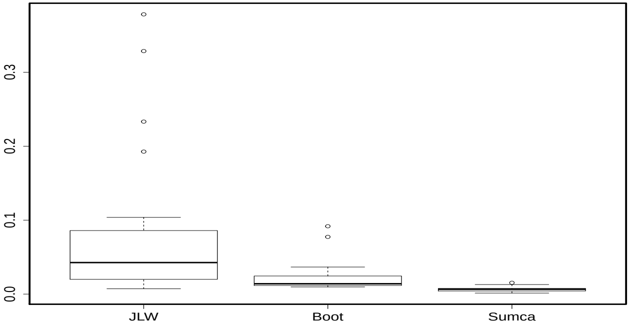
$v_i \overset{\text{i.i.d.}}{\sim} N(0, A); p_{ij} = \mathrm{P}(y_{ij} = 1|v_i);$

$y_{ij} = 1$ or 0: whether or not the jth individual in the ith small domain does not have health insurance;

$x_{ij1}, x_{ij2}, x_{ij3}$: family size, educational level, and total family income of the jth unit in the ith small domain, respectively

Health Insurance of Minority Sub-populations Data

- Figure 4: Boxplots of square roots of MSPE estimates for health insurance data using JLW, bootstrap, and Sumca methods



COVID-19 Pandemic in Manitoba, Canada

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- We provide some summary statistics about the COVID-19 pandemic in Manitoba including daily number of infected cases, number of deaths, number of recovered, and also number of infected cases based on age-sex, and health regions in Manitoba from March 12 to July 24, 2020.

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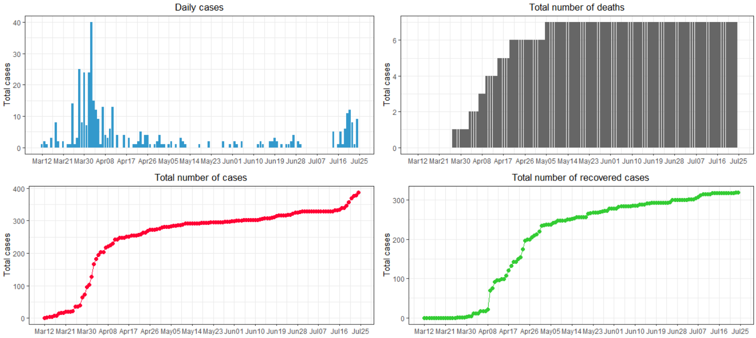
- We provide some summary statistics about the COVID-19 pandemic in Manitoba including daily number of infected cases, number of deaths, number of recovered, and also number of infected cases based on age-sex, and health regions in Manitoba from March 12 to July 24, 2020.
- The total number of domains (m) for this analysis is $80(= 8 \times 2 \times 5)$ based on age $\times$ sex $\times$ health region.

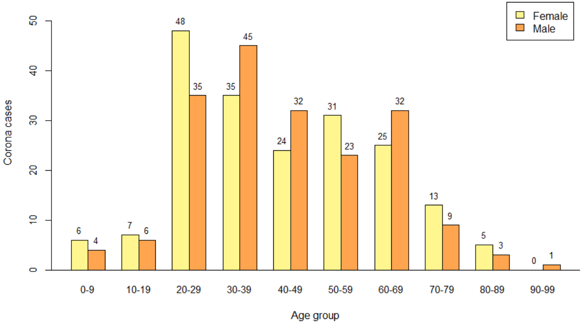
COVID-19 Pandemic in Manitoba, Canada

- Figure 5: Plots of COVID-19 daily cases, total number of infected cases, total number of deaths, and total number of recovered cases in Manitoba from March 12 to July 24, 2020.



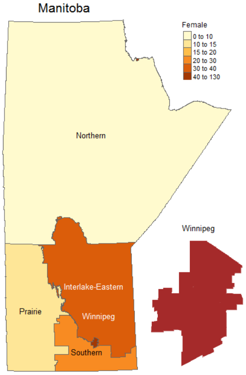
COVID-19 Pandemic in Manitoba, Canada

- Figure 6: Histogram of COVID-19 cases stratified by age and sex in Manitoba from March 12 to July 24, 2020

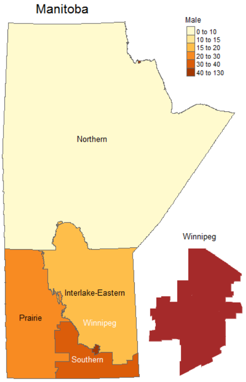


COVID-19 Pandemic in Manitoba, Canada

- Figure 7: Maps of COVID-19 cases by sex and health regions in Manitoba from March 12 to July 24, 2020



(a) Female



(b) Male

COVID-19 Pandemic in Manitoba, Canada

- We consider the following Poisson model:

$$\log(\theta_i) = \log(E_i) + \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + v_i, (i = 1, ..., 80),$$

where

$v_i \overset{\text{i.i.d.}}{\sim} N(0, A); \theta_i = E(y_i|v_i)$;

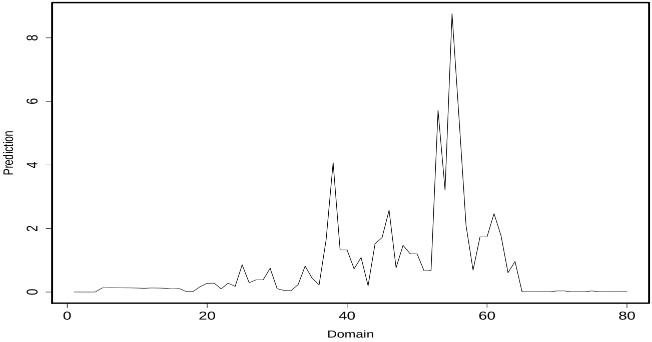
y_i: number of infected people in the ith small domain;

E_i: expected number of infected people in the ith small domain adjusted by the population size;

x_{i1}, x_{i2}: proportion of immigrants and Indigenous people in the ith small domain, respectively

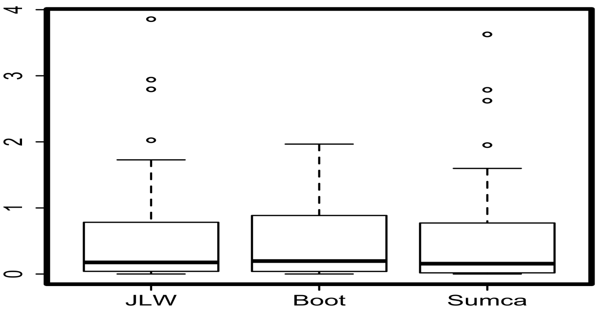
COVID-19 Pandemic in Manitoba, Canada

- Figure 8: Prediction of average rate of infected people for 80 domains (age-sex-health region) in Manitoba from March 12 to July 24, 2020



COVID-19 Pandemic in Manitoba, Canada

- Figure 9: Boxplots of square roots of MSPE estimates of prediction of average rate of infected people using JLW, bootstrap, and Sumca methods



Conclusions

- There is a significant computational advantage of Sumca estimator compared to existing resampling methods in SAE:

  • Double bootstrap approach is computationally very intensive.

  • McJack method requires the Monte-Carlo sample size, K, to satisfy $m^2/K \to 0$.

Conclusions

- Although, for the second-order unbiasedness of the Sumca estimator, there is no restriction on K (in theory), a larger K would help to reduce the variation of the MSPE estimator.

Conclusions

- Although, for the second-order unbiasedness of the Sumca estimator, there is no restriction on K (in theory), a larger K would help to reduce the variation of the MSPE estimator.
- A practical recommendation is to choose a conveniently large K as long as computational burden is not a concern. In all of our simulation studies and real data analyses, we have chosen $K = m$ to ensure that the results are accurate.

- Questions?