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J. Mohn (1838–1882)

A. N. Kiaer (1838–1919)

Early Scandinavian Contributions to Survey Sampling

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Abstract

This paper is concerned with the developments of the representative method from its suggestion by Kiaer in 1895 until its acceptance at the International Statistical Institute meeting in Bern 1925. The focus is on contributions from the Scandinavian countries while not intending for a full account of all work made during the 1895–1925 period. A main aim is to provide plausible explanations to the developments of the method with respect to applications in official statistics. Relating to the non-sampling errors facing National Statistical Institutes in our time, it is relevant to ask if we would have been better off today if our methods would have stayed within the original methodology of the representative method.

Keywords: the representative method, random selection, purposive selection, official statistics.

1 Introduction

The initial idea was to write a paper covering Scandinavian contributions to survey sampling from the presentation by Anders Kiaer at the 1895 International Statistical Institute (ISI) meeting in Bern up to the 1980s. When collecting information on contributions three vital curiosities emerged. The first was the domination of contributions from Sweden from the 1950s onward. One explanation might be how the reference search was conducted. Partly this may be so, but publications from the 1950s and onward are relatively well documented in different data bases.

The second curiosity was the late adaption of the Neyman (1934) theory. Statistics Sweden started employing sample surveys in the 1950s, but probability sampling were rarely used in the beginning. The theory was generally known and also suggested by users of official statistics. The third curiosity was the seemingly inactivity in developments of the representative method up to its acceptance at the 1925 ISI meeting.

It is a cliché but it is true, understanding the present is not possible without knowing the history. This is also true with statistics. An example is the way National Statistical Institutes (NSI) are publishing survey statistics, i. e. point estimate plus/minus a margin of error. This has historical roots and users may in those days have been able to correctly interpret the statistics. With the wide use of official statistics today it is reasonable to assume that alternative ways of publications of statistics are required for correct interpretation.

The purpose of this paper is therefore to study early developments of survey sampling, with a focus on contributions from the Scandinavian countries. The aim is to obtain a picture of the developments from Kiaer's suggestion in 1895 until its acceptance at the 1925 ISI meeting.

Kiaer's contributions are given attention in the next section, followed by a section describing applications of the representative method in the period. Section 4 includes some methodological issues and a discussion is saved for the final section.

2 Kiaer and the Representative Method

The importance of the contributions made by the Norwegian statistician Anders Kiaer (1838–1919) for the development of the survey sampling methodology cannot be understated. The idea of taking a sample instead of a census was presented by Kiaer at the ISI meeting in Bern 1895; he called it the "representative method". There he suggested selection of samples giving miniature representations of the populations as a complement to censuses. The arguments were lower costs and faster dissemination of results.

His importance for the development of sample surveys is demonstrated by the many sources describing his work and efforts. One is the Encyclopedia of Mathematics (Kiaer, Undated), another is Statisticians of the Centuries (Heyde and Seneta, 2001). A third source is Kruskal and Mosteller (1980) which gives a thorough description of Kiaer's struggle with the ISI to have the representative method accepted. A story different from the one generally told, on Kiaer's role in developing the representative method idea, is given by Lie (2002).

For increased precision (reduced bias error) of sample statistics, Kiaer and his colleagues at the Norwegian statistical bureau used sampling designs involving e. g. stratified sampling, cluster sampling and its special case of systematic sampling. They used auxiliary census data in evaluation of sample representativeness. These basic methodologies are found in later applications of the representative method and they are standard tools in survey sampling design of today.

Kiaer did not use random sampling, but Kiaer (1897)[p. 39] argues the selection of units made are as if they would have been drawn by lots, i. e. the sample can be treated as a random one. Jensen (1925b)[p. 548] supports this view considering a two-stage sampling design with purposive selection in the first stage of the social survey. In the second stage, men of specific ages were considered and within those age categories men with names starting on a set of specific letters are selected to the sample. This second sampling stage is by Jensen considered as random sampling.

Kruskal and Mosteller (1980) do not classify Kiaer's sample selection methods as random sampling, in the meaning of a probability sample. The earliest known application of random sampling is, according to Kruskal and Mosteller (1980), a study of housing conditions in Gothenburg around 1910. (The same survey is in Jensen (1925b) dated to 1911.) The survey was conducted by K. A. Edin (1880–1937) and reported in Edin (1912). The sample selection procedure is described in Dalenius (1957)[p. 40].

Lie (2002) draws attention to three studies using samples instead of censuses conducted prior to Kiaer's application of the representative method. According to Lie, the first study was made at the Norwegian State's statistical office of which Kiaer was the head. The design of the survey, aiming for statistics on agricultural production, was made by Jacob Mohn (1838–1882) and conducted by Mohn and Boye Strøm (1847–1930) around 1875. Both Mohn and Strøm were colleagues with Kiaer. The second study was conducted in the 1870s by Mohn. The third, a survey on household consumption in 1888, was conducted by Strøm.

The last survey was undertaken at the Norwegian Central Bureau of Statistics (CBS), formed in 1876

by reorganization of the state statistical office. Kiaer was appointed director at the CBS in 1877. Thus, prior to Kiaer's own study in 1897, his colleagues had several times employed the basic idea of choosing a sample. This has to be kept in mind when attributing the start of survey sampling to Kiaer and his representative method. According to Lie it is reasonable to consider Mohn's work in the mid 1870s as the first step of the development of the representative method.

Another enlightening story in Lie (2002) provides an answer to a question raised by Kruskal and Mosteller (1980). They describe the 1903 ISI meeting as the last one Kiaer promoted the representative method. Thereafter it was not brought up again until the 1925 meeting, six years after the death of Kiaer. Kiaer seems not to have brought up the topic elsewhere either. So, why was not the representative method on the agenda between 1903 and 1925? Lie gives an explanation.

Kiaer was not a mathematician while probability calculations could be done by treating the sample as if it were taken at random. Mathematicians did such calculations and sometimes they implied that Kiaer's survey results were questionable. Kiaer could not respond and explain the differences. The most damaging example described by Lie, is an estimate of the number of disabled people in Norway in 1906. With probability calculations and by scrutinizing the survey design, the critics claimed Kiaer's estimate was too low. Kiaer did a complementary survey showing the critics were right! Kiaer's potentially erroneous estimate may not had been such a deal if it was not for its purpose. In this case it was to be used for decisions on a general disability insurance. The insurance plan was controversial and publicly debated.

The story on the distrust and public discussion on Kiaer's survey results may explain why Edin (1912) chose to use random sampling by lot (stratified simple random sampling). Dalenius (1957) gives the following citation of Edin:

> *The main object of the procedure here described has naturally been that no one could possibly have the slightest reason for saying that the sample was biased, or on the whole, for whatever reason, that preferably worse apartments have been included in the survey.*

3 The Representative Method in Practice

Adolph Jensen (1866–1948) from Denmark was a member (Rapporteur) of the ISI committee preparing the report on the representative method for the ISI meeting in 1925 (Jensen, 1925a). In an appendix to the report (Jensen, 1925b) he fulfilled a decision made by the ISI already in 1903; a report on the practice of the representative method. The reports give valuable insights on the state-of-the art at that time and the earlier evolution of the representative method.

Jensen (1925b) reports on 50 studies between the late 1890s up to the early 1920s. Over this period he found a pattern where the representative method where applied to some extent around the time Kiaer was promoting the method. Thereafter a decline in interest of the method up to the mid 1910s was indicated. World War I (WWI) seems to have fueled an increasing interest in the representative method followed by an even higher interest after the war. Jensen concluded the driving force behind the increasing number of applications was the growing demand of statistics paired with limited resources for statistics production. Notably, many of the earliest studies comprise of selection of samples from census data. Apart from experimentation these were motivated by timeliness and resource limitations. Instead of analyzing all data from a census, sample results could be disseminated faster to a lower cost.

Jensen's survey covers studies made in 13 different countries with a tendency of a center of gravity towards the Scandinavian countries. He found this not to be a surprising result even when taking into account he himself was Danish. One explanation given is that representative sampling is a Norwegian idea and may have had a particular influence on the methods used in the neighboring countries,

Denmark and Sweden. His explanation can further be supported by the long history shared by these three countries.

Jensen's paper is divided into sections by how the sample is selected. Two of the sections deal with random sampling of units and random sampling of groups, respectively. Around half of the studies reported on are classified into these two categories of sample selection. However, it seems the meaning of a random sample was that the sample could be treated as a random sample. Many studies reported as using random sampling involve systematic sampling but it does not seem the starting points were selected at random. The only study, where the random sampling can be interpreted in the meaning of probability sampling, is the earlier mentioned Edin (1912).

Other sections in Jensen's paper cover applications of purposive sampling. In one section, two studies are considered with purposive selection of groups and random selection of units in selected groups. These are, of course, examples of two-stage cluster sampling, although not using probability sampling in the two stages.

Stratified sampling is a frequently applied sampling design in the studies reported, a design also applied by Kiaer. This is obviously necessary in purposive sampling where the ambition is to have a sample representing a miniature of the population. One stage cluster sampling is reported in several studies where the purposes were to sample from census data or registers to reduce time and costs in calculation of statistics. Some examples are experiments to test the representative method.

Upon reading the report for the ISI 1925 meeting (Jensen, 1925a) it is striking how close the reasoning on and recommendations for designs of representative sampling is to modern theory and practice. These do not include probability sampling. However, on page 487 the following is stated regarding the desire to avoid sample data being "one-sidedly coloured":

> *The handiest method would seem to be that the inquiry is made according to some mechanical principle or other which is unconnected with the subject and purpose of the inquiry, with essential condition that every unit in the population or universe in question shall have an equal chance of inclusion in the sample.*

Later on Jensen suggests to draw the sample by lot if there is no "mechanic principle", e. g. systematic selection, deemed to yield a representative sample.

4 Methodology

A discussion of the two meanings of statistics, statistics and statistical methodology (mathematics), seems to have emerged during the 1910s when interpreting Edin (1916). Kiaer's idéa of taking a sample enabled studies of the society in new areas and in more detail. Perhaps because of the method being new, Edin (1916) suggests prioritizing, for a time ahead, a deeper understanding of the statistics rather developing new mathematical formulas for "fine tuning" of statistics.

This implied division among statisticians may have been much deeper when considering the general use of statistics. The Danish statistician Harlad Westergaard (1853–1936) has the the following paragraph in Westergaard (1916).

> *Still it is a fact, that there are at present not one, but several corps of statisticians, each trying earnestly to promote the science, but hardly able to cooperate for lack of mutual sympathy and sometimes acting in direct opposition to one another.*

In the paper and in Westergaard (1918) he promotes use of simplicity over complexity. There was a trend towards finding models or formulas giving more universal relations between variables. In many cases simple tabulation of data would do equally well or better than correlation calculations and fitted

models. Notably he wrote about the importance of keeping calculated statistics close to the original data.

The first volume of the statistical journal *Nordisk Statistisk Tidskrift* was published in 1922. In 1929 the first volume of an English edition, *Nordic Statistical Journal*, was published. The founder was Thor Andersson (1869–1935) and the last volumes of both journal editions were published in 1932. The journal included papers on a variety of aspects on statistics. Some of the contributions were concerned on problems related to the representative method.

One contribution in the journal is Jensen (1923) who proposed some labor saving methods in statistics. Statistical agencies could not with existing resources meet the increasing demand of new statistics. One argument of his was that statistical agencies took on too big tasks than necessary. An example is the use of one and the same questionnaire to all units in the population, while often a shorter version would do for most units. Another argument is time placed on calculation of too detailed statistics.

Jensen also brings up the idea of the representative method. He points to it being put on the table by Kiaer for almost 30 years ago. The initial skepticism towards the method he believes to be grounded in the fear of unprofessional applications. This skepticism can be understood, but Jensen means it cannot be an argument today. There are enough of examples of applications of the representative method yielding satisfactory results.

Another author with several contributions in the two journals is Tor Jerneman (1897–1965). His dissertation (Jerneman, 1931) includes a number of interesting statements. In the first paragraph of the paper he states he prefer to use the English word “sampling” instead of the Swedish “den representativa metoden”. He finds the Swedish name misleading and rather cumbersome.

In the second paragraph he motivates the use of sampling for reduction of response burden.

Now, one of Jerneman’s contributions is a derivation of the standard error for the sample mean under SRS:

$$\sigma_n = \frac{1}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}\sigma. \tag{1}$$

Here N and n denotes population and sample sizes, respectively, and σ denotes the population standard deviation of population values.

He notes A. A. Tschuprow (1874–1926) had much earlier in Tschuprow (1918) claimed to be the first to derive this expression.

Jerneman realizes replacing the unknown σ with a corresponding sample value (σ_s) will either give a higher or a lower value. To protect against an underestimation of the standard error he suggests the alternative value

$$\sigma_n = \frac{1}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}\sqrt{\sigma_s^2 + \bar{\sigma}^2} \tag{2}$$

where $\bar{\sigma}$ is an expression for the standard error of σ_s^2.

5 Discussion

It is accurate to think of this paper as merely a prestudy or a qualitative study upon which hypotheses can be formulated on the early developments of survey methods. Here focus has been on Scandinavia where, of course, developments were integrated with those elsewhere. The report to the ISI in 1925, for example, was based on the work of a six member committee of which one were from the Nordic countries. There, Jensen’s survey of applications covered 13 countries, so the topic of

the representative method was of a world wide interest. Furthermore, the names and contributions cited here cannot be claimed to cover all the important contributions from the Scandinavian countries either.

However, based on what is included, one can formulate a number of hypotheses causing the evolution of the survey methodology in the years between Kiaer and Neyman. Production of official statistics is not made in isolation. There is a production organization and there are users and stakeholders. So there is a whole environment to convince of the pros of a new methodology. Going from a census to a sample survey would also imply the need of education in interpretion of statistical results. This is likely one part of the explanation of the early skepticism at the ISI meetings. Members would have to consider their organizations and users, and they might themselves have dual interests.

WWI was a mediator forcing statistical agencies to produce new statistics in a short time with limited budgets. Several studies around 1915 reported on by Jensen (1925b) consider countries food production and consumption, which were important war time issues. The postwar time further demanded new statistics with probably less resources in most European countries.

With the representative method being accepted, by necessity, in the early 1920s, the problem of quantifying the estimation error gained interest among mathematicians. It seems plausible the statisticians interpreted the inference problem differently from the mathematicians. The latter introducing randomness saw a range of possible values on the statistic out of which one would be realized. On the other hand statisticians had gone from censuses, providing "true" population values, to selection of a sample in such a way the sample based values are close to the ones in the population. Remember that stratification was used by Kiaer to account for differences among groups of the population in order to obtain a more correct value. It was not used for a reduction in standard errors of estimates.

A similar explanation can be placed on why systematic samples without randomization of the first unit are interpreted as random samples. Of the around 25 studies using random selection reported by Jensen (1925b), only one study used proper randomization. An interpretation is that if the selection of a cluster matters, your survey design has excluded important factors. Thus, with a design taking into account of all the important factors affecting what you are studying, the selection of a cluster only marginally affects your result. Without having considered the period after Neyman (1934), one can foresee an even further distance between statisticians and mathematicians with regard to statistics.

It is interesting to note that non-sampling errors were of concern already from the beginning. Non-response was an issue in several studies reported by Jensen (1925b). This problem has drastically increased, particularly in household surveys, and today in 2024 the levels of non-response threatens validity of statistics and drains NSI's resources. The trend is to abandon traditional sample surveys and adapt new approaches to produce statistics.

After Neyman (1934) mathematical statisticians were spurred to further develop the theory and have made important contributions with applications in most research areas. These developments of the theory generally assumes full response. There is also a vast literature on estimation under survey non-response. However, as stated in the following quote by Brick (2013) on the literature on non-response,

> *the central problem, in our opinion, is that even after decades of research on nonresponse we remain woefully ignorant of the causes of nonresponse at a profound level.*

This quote begs the question: With regard to household surveys, would survey statisticians of today have been better off if the representative method had stayed within the methodology of Kiaer?

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