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A Tribute to the Centenarian Statistician C. R. Rao

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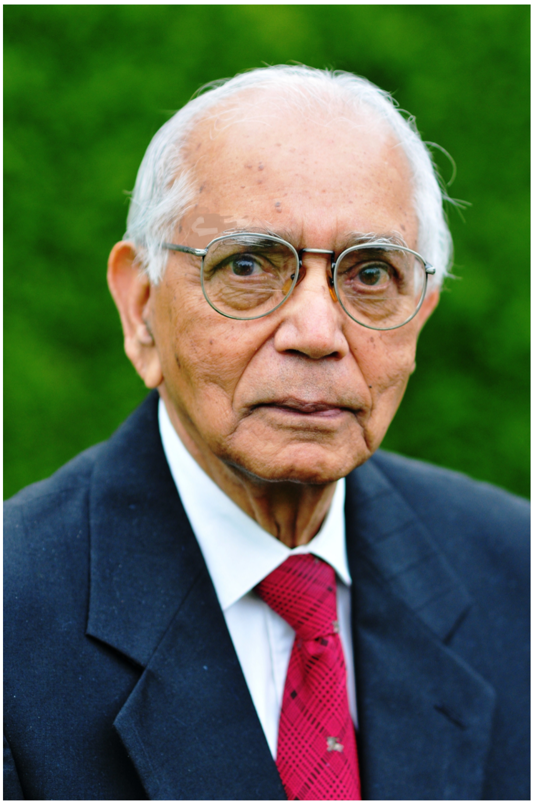
Abstract

Professor Calyampudi Radhakrishna Rao, (fondly known as Dr. Rao to his students and CR to his professional colleagues in US) was born on September 10, 1920 and died on August 22, 2023, at his daughter's place in Buffalo, New York just 18 days before his 103rd birthday. He was a legendary statistician, born along with modern mathematical statistics. He had the unique distinction of being one of three distinguished centenarian statisticians who lived in the century of the birth of the discipline and the century of its maturity, he having contributed immensely for its maturity. C. R. Rao adored statistics and adorns statistics. Here is a tribute to him.

Keywords: Cramer-Rao inequality, Rao-Blackwellization, Rao Distance, Information Geometry, Econometrics, Score Test



1 Introduction

The statistics legend C. R. Rao (Calyampudi Radhakrishna Rao) who lived a glorious life of 102 years passed away on August 22, 2023, at his daughter's home in Buffalo, New York. He was born in India on September 10, 1920.

Dr. Rao was always at the frontiers of statistics knowledge, and often those frontiers chased him as he stretched those frontiers. He communicated a paper to the *Proceedings of the National Academy of Sciences* in his 100th year (2020) (Baisuo Jin, C. R. Rao, Yuehua Wu, and Li Hou, "Estimation and model selection in general spatial dynamic panel data models", *Proceedings of the US National Academy of Sciences*). He visualized the role of artificial Intelligence and machine intelligence as far back as 1969 (C. R. Rao, Computers and the Future of Human Society, Statistical Learning Society) that are just now being explored. Rao co-edited a volume of *Handbook of Statistics*, Vol. 49, titled *Artificial Intelligence* for Elsevier in 2023 (with S. G. Krantz and Arni Rao (the co-author of this tribute)). C. R. Rao was the series editor of *Handbook of Statistics* series for more than three decades and coedited several volumes.

Dr. Rao co-authored a book on artificial intelligence in his 100th year (2020) with Pereira and Oliveira (Pereira, Basilio de Braganca, C. R. Rao, and Fabio Borges de Oliveira, *Statistical Learning Using Neural Networks*, C. R. C. Press, Chapman, and Hall). We three, the authors of this article, are privileged to be associated with him as his students at Indian Statistical Institute (I. S. I.) or collaborators. Speaking of his teaching, he would ask his students, by choosing one student at random in each class, to go to the blackboard and derive from a practical example a very useful statistical result-such as the best estimator of a parameter, and in a later class prove the same result using the Rao-Blackwell theorem. In the afternoon the same result or similar ones are worked out with a real life problem with data. Quite often he used an interesting example with a joke and with his characteristic chuckle and smile. He made his students, several hundreds of them, feel nostalgic in later years about the teaching at the Research and Training School of the Indian Statistical Institute that he oversaw. See obituaries on C. R. Rao published in *Nature* (2023 by Peddada and Khattree), in *Science* (2023 by Banks and Clarke), and in the *IMS Bulletin* for his role in elevating the Indian Statistical Institute and the official Statistical Systems in India.

C. R. Rao was born around the same time the fundamental concepts of sufficiency, efficiency, and likelihood were introduced by Fisher. Twenty years later Rao studied statistics under the tutelage of Professor P. C. Mahalanobis with the dictum that "statistics has a purpose", and exploited these concepts of sufficiency, efficiency, and likelihood and used them for good purpose, thus making breakthroughs in statistics. He had keen interest in all branches of knowledge where statistical thinking is useful. This is the hallmark of the Indian school of statistics developed by Mahalanobis and Rao over fifty years (1930-1980). Statistics was developed in India as a core subject surrounded by various branches of science, with the Banyan tree serving as the logo. Dr. Rao developed statistical theories and methods from practical problems and applied them to a variety of disciplines such as anthropometry, biometry, psychometry, econometrics, engineering, and environmental science. That is why the citation of the US Medal of Science to Dr. Rao, awarded by President Bush in 2002, stated "...... for his pioneering contributions to the foundations of statistical theory and multivariate statistical methodology and their applications, enriching the physical biological, mathematical, economic and engineering sciences ...". Fisher founded the Biometric Society in 1947 (when Rao was his Ph. D. student) with five regional Chapters, comprising of Britain, France, Australia, United States, and India. The Indian Region of the Biometric Society was established with P. C. Mahalanobis, who received the coveted Weldon medal for his work on biometry from the University of Oxford in 1944 as its President, and C. R. Rao as the Secretary. Rao would later become the President and Honorary Life Member of the Biometric Society.

C. R. Rao was a brilliant student of mathematics since the beginning (*Significance* 2020 and *Significance*, 2012). He had received very good training in high school to master's in mathematics at Andhra University, India. According to the MR database of the American Mathematical Society, C. R. Rao published 19 articles during 1941-1945 from Kolkata, India before he went to pursue a Ph. D. in Cambridge, England. Before his groundbreaking article of 1945 titled "Information and the accuracy attainable in the estimation of statistical parameters" which was published in the *Bulletin of Calcutta Mathematical Society*, Rao worked on topics like general probability, combinatorial designs, characterization of probability distributions, problems in number theory, etc., C. R. Rao completed Ph. D. in 1948 from Cambridge under the supervision of R. A. Fisher and returned to Indian Statistical Institute, Kolkata as a Professor, a position created to him by P. C. Mahalanobis, the founder Director of the Institute. Rao lead teaching and research activities at I. S. I. for three decades. Speaking to an audience at I. S. I., R. A. Fisher remarked that at one time most of the professionally trained statisticians in the world were from India and they were all trained by Rao. After retiring from I. S. I., Rao moved to US and spent three more decades leading teaching and research in statistics, and multivariate statistics, in particular at the University of Pittsburgh and the Pennsylvania State University.

2 The impact Rao's Work on Various Branches of Sciences and Engineering

We start by describing Rao's path-breaking contribution of introducing the concept of differential geometry in statistics and deriving the all too powerful result-the Cramer-Rao inequality.

2.1 Rao's work prior to 1948

Problems arising out of geometrical thinking have been of interest to Rao since the beginning. For example, in the article in 1941 published in *The Mathematics Student*, using a method to compute the volume of a prismoid in $n-$space, Rao showed how the geometrical probability functions can be treated as useful tools. In the article published in 1944 in *Science and Culture*, Rao developed a method of analysis of experimental block design where an experiment in randomized blocks is not independent of each other. In a series of articles in 1945 published in *Sankhya* and *Science and Culture*, Rao considered Markov's theorem on matrices and generalized it to make it flexible to test linear hypotheses. He also considered the equality of the means of a $p-$variate normal population whose covariance matrices are unknown. Rao alone and also along with R. C. Bose and S. Chowla in a series of articles published in 1945 in journals: *Bulletin of Calcutta Mathematical Society, Proceedings of National Academy Sciences, India Section A, Proceedings of National Academy Sciences, India, Proceedings of Lahore Philosophical Society*, etc., studied a variety of problems arising in Galois fields, theory of congruences, quadratic functions of the type $x^2 + ax + b\,(\bmod\, p)$, where p is an odd prime, a and b are arbitrary integers.

For some integer c, the integral order of $x^2 + ax + b\ (\bmod\, p)$ is defined as the least positive integer n such that

$$x^n \cong c\left(\bmod\ p, x^2 + ax + b\right). \tag{1}$$

By the age of 25, Rao published more than 18 articles including the monumental article mentioned at the beginning of this tribute that helped create a solid foundation for modern statistical science through Cramér–Rao inequality, and Rao–Blackwell theorem. This article also created a new subject area called information geometry. We will describe the basics of the 1945 *Bulletin of Calcutta Mathematical Society* article of Rao and some necessary technical elements of it in the following paragraphs. Let

$$\sum_{n=0}^{\infty} a_n \quad \text{and} \quad \sum_{n=0}^{\infty} b_n$$

be two convergent series such that

$$\sum_{n=0}^{\infty} a_n \geq \sum_{n=0}^{\infty} b_n,$$

then

$$\sum_{n=0}^{\infty} a_n \log \frac{b_n}{a_n} \leq 0. \tag{2}$$

In the inequality (2), a lot of information is stored, and based on the values of the sequence of terms of a_n and b_n, the future events can be predicted. The questions like how much uncertainty can be born in the future based on random variables defined in a space or the level of uncertainty that can be allowed based on the sequence of random variables within a space can be explained using the above kind of inequality or in its continuous equivalent. Fisher information measure, say, $\mathcal{F}$ on a parameter θ contained in a random variable X (in some space S) having continuous probability density $\phi(.,\theta)$ with $\sigma-$finite measure ν is defined by

$$\mathcal{F}(\theta) = \int_{-\infty}^{\infty} \frac{\partial^2 \text{log}\phi(X,\theta)}{\partial\theta^2}\phi(X,\theta)dx. \tag{3}$$

The probability density $\phi(X,\theta)$ is assumed to be differentiable with respect to θ for any measurable set $C \subset S$, and

$$E\left(\frac{\partial \text{log}\phi}{\partial\theta}\right)^2 = \int_{-\infty}^{\infty} \frac{\partial^2 \text{log}\phi(X,\theta)}{\partial\theta^2}\phi(X,\theta)dx.$$

C. R. Rao in his 1945 ground-breaking article asked the question "What do we mean and know by the information on an unknown parameter θ?" and systematically answered it. He explains that the extent to which uncertainty regarding the unknown value of θ is reduced as a consequence of a prior observed value of X.

Suppose n sample observations $x_1, ..., x_n$ with a parameter θ are estimated by the function $t = f(x_1, ..., x_n)$, then Rao showed that

$$\text{Var}(t) \not< \frac{1}{I}, \tag{4}$$

where Var(t) is the variance of t, and $I =$ Var$\left(\frac{1}{\phi}\frac{d\phi}{d\theta}\right)$= E$\left(-\frac{\partial^2 \text{log}\phi}{\partial\theta^2}\right)$. Rao further considered information matrix with several unknown parameters $\theta_1, ..., \theta_q$ with probability density function $\phi(X, \theta_1, ..., \theta_q)$. Suppose $t_1, ..., t_q$ be the estimates of $\theta_1, ..., \theta_q$ with the joint distribution $\Phi(t_1, ..., t_q; \theta_1, ..., \theta_q)$, then the information matrix on $\theta_1, ..., \theta_q$ due to $t_1, ..., t_q$ is defined by Rao as $\|F_{ij}\|$, where

$$F_{ij} = E\left[-\frac{\partial^2 \text{log}\Phi}{\partial\theta_i\partial\theta_j}\right]. \tag{5}$$

Further, Rao considered distance between two populations (which is now popularly known as Rao distance) with two sets of parameters $\theta_1, ..., \theta_q$ and $\theta_1 + \delta\theta_1, ..., \theta_q + \delta\theta_q$. Using Riemanian geometry, Rao showed that the geodesic distance between these two populations as

$$ds^2 = \sum\sum g_{ij}\delta\theta_i\delta\theta_j, \tag{6}$$

where

$$g_{ij} = E\left[\left(\frac{1}{\phi}\frac{d\phi}{d\theta_i}\right)\left(\frac{1}{\phi}\frac{d\phi}{d\theta_j}\right)\right]. \tag{7}$$

Arni Rao and Steven G. Krantz extended the Rao distances to complex planes by using conformal mapping principles of complex planes. In a series of collaborative articles (Arni Rao, *Handbook of Statistics*: *Geometry and Statistics*, Vol. 46, pp: 401-464, Steven Krantz and Arni Rao, *Handbook of Statistics*: *Geometry and Statistics*, Vol. 46, pp: 3-19, Arni Rao and Steven Krantz, *Handbook of Statistics: Information Geometry*, Vol. 45, pp: 43-56, Arni Rao and Steven Krantz, *Cell Patterns*, 2020) they showed that such extensions have implications for virtual tourism technologies.

2.2 Rao's work after 1948

During 1948-2023, C. R. Rao developed individually and also through collaborative efforts, various statistical methods, and theories, which include linear statistical inference, multivariate analysis, combinatorial designs, information geometry, orthogonal arrays, M-estimation, second-order efficiency, characterization of distributions, Minimum Norm Quadratic Unbiased Estimation (MINQUE) theory, random coefficient modeling, and matrix theory and their applications in statistical inference (for example, see, Efron B. et al., *Significance,* 2020, Andrews G. E. et al. *Notices of the AMS,* 2022, Khattree, R. et al. *Bulletin of the IMS,* 2023, etc.).

C. R. Rao published more than 15 books and over 475 research articles throughout his research career spanning over 80 years. His work impacted research in other fields such as computer science, especially in signal processing, physics, information geometry, biology, anthropology, economics, etc. He was conferred an Honorary membership of IEEE for his pioneering contribution to signal processing through his Cramer-Rao inequality. Apart from receiving various prizes and medals Rao was honored with 40 honorary doctorates from various countries. He was also elected as the President of several statistical societies. .He was elected as a member of several national academies of science, including the US Academy of Science, Indian Academy of Science, the Third World Academy of Science.

Cramer-Rao inequality is used as an industry standard for noise reduction in communications and signal processing for which IEEE honored Rao with a distinguished Honorary Membership of IEEE during the platinum jubilee year of its Signal Processing Society. The orthogonal designs developed by Rao constitute the engineering designs to enhance productivity promoted by Taguchi. Cramer-Rao bound is used as Quantum Cramer-Rao bound in quantum physics. Almost all existing theories of physics, and some new ones, were derived from a principle of reaching a suitably defined Cramer-Rao bound (Frieden (2004)). Rao's score test, Fisher-Rao distance, MINQUE theory, his concept of second order efficiency, etc. are examples of his contributions of lasting value.

3 The impact of Rao's work on Econometrics and Other Social Sciences

3.1 Rao's Contributions to Econometrics

In this section we examine, in a bird's eye view, the work Rao did in the field of social sciences and the impact it had on those disciplines. Mahalanobis and Rao initiated econometrics research in India by organizing a two-day session of the international econometric society meetings in India in December of 1951, the same year Mahalanobis was elected a Fellow of the International Econometric Society in recognition of his work on large scale sample surveys and National Income Statistics for the United Nations Statistical Commission. A decade later in 1960 Rao organized a special session in the Indian Science Congress on the applications of mathematics and statistics to economics. The same year he and N. S. Iyengar organized the first Indian Econometric Conference at the Indian Statistical Institute. Rao was the Founder President of the Indian Econometric Society (1970-75), and he was elected the Fellow of the International Econometric Society in 1972. Rao organized a Workshop on the Database of the Indian economy and helped Mahalanobis establish the Central Statistical Organization and State Statistical Bureaus for providing reliable economic statistics needed for national planning. For the first time in the history of the world, statistics and operations research were employed, through the active participation of an academic institute, to prepare economic plans for development affecting the lives of millions of people. Rao played a significant role in this effort, along with Mahalanobis.

Econometrics deals with both observed variables (price index, current income, etc.) and unobserved conceptual and or subjectively defined variables (expected price, lifetime income, etc.). Econometrics appears mainly in three forms: i) as a system of independent equations in which both errors in variables and errors in equations appear (1926-1944), ii) as a system of independent equation with errors in equations but not in variables (1944-) and iii) as a system of equations in observables and non-observables (factor analysis). Rao made fundamental contributions to statistical methodology needed in each of these three different approaches. Ragnar Frisch, who founded and pioneered the development of econometrics as a new discipline, conceived it as a confluence of mathematics, economics, and statistics whose objective it was to understand and tame the economic environment. A paper by Frisch and Mudgett published in 1931 (Ragnar Frisch and Bruce D. Mudgett "Statistical Correlation and the Theory of Cluster Types", *Journal of American Statistical Association*) is an excellent exposition of his intuitive conceptualization of statistical modeling in economics, on how to use sample information to impose parametric restrictions to identify and estimate linear models. Visualization of clustering of sample observations could suggest what parametric restrictions characterize that cluster. It is this duality between the parametric and sample spaces, given an assumed probability distribution (and the likelihood function), that characterized the inferential work of Rao using differential geometric concepts. Thus, Rao was both a Fisherian and a Frishian.

Frisch interpreted economic variables as observed and unobserved, and as endogenous, and exogenous. Endogenous variables are those that are explained by the economic model of the economic system, and exogenous variables are variables that are determined outside the economic model and affect the variables in the economic system but are not affected by them. An econometric model is a simultaneous system of as many equations as there are endogenous variables that impose constraints on the free movement of variables in a finite-dimensional space. Thus this system gives rise to the correlation between variables on restricted hyperplanes or hyper surfaces rise to the correlation between variables and the confluence of the variables on restricted hyperplanes or hyper surfaces.

In connection with the above errors in variables model, used as an exploratory model to discover economic laws, Frisch posed the following question to Neyman at the 1936 European meetings of the Econometric Society: if two endogenous variables are linearly related to an exogenous variable measured with error by two equations with errors in equations, under what distributional assumptions on the errors in variables and errors in the two equations does there exist a linear regression relation between the two endogenous variables. The motivation for this question was to determine in a very simple two equation model under what conditions a structural equation (with a possible economic interpretation) exists between two endogenous variables with one exogenous variable. A complete answer to this question was provided by Rao just six years later in 1943 in his Master's thesis (as cited in C. R. Rao "Note on a problem of Ragnar Frisch", *Econometrica*, Vol. 15, 1947). In 2021 as a birth centennial tribute to Rao, Prakasa Rao and Kumar extended this result of Rao (*Journal of Quantitative Economics*, Vol. 19, 2021) by finding conditions under which two endogenous variables linearly related to two exogenous variables, measured with errors (and hence stochastic) would admit two independent linear regressions between the two endogenous variables. This is a typical model of a partial market equilibrium for a commodity as a system of two simultaneous equations, one a demand equation and the other a supply equation. Thus, Rao can be regarded as one of the pioneers in the development of econometric theory formulated by Frisch with both errors in variables and errors in equations. While this line of research proposed by Frisch with errors in variables was abandoned by econometricians, Rao followed this line of work and developed a new field within statistics with the name characterization of probability distributions. The application of this method to econometrics, however, remained underexplored.

The Cowles Commission for Research in Economics developed econometric theory dealing with identification and consistent estimation of a system of linear econometric relations employing likelihood and quasi-likelihood methods. In doing so it used models with no errors in variables and depended on the prevailing theories of statistical inference in linear models, not recognizing that most of that work was done by one person, C. R. Rao. The econometricians' concept of identification is nothing but a special case of the estimability concept developed by Rao (when the sample size is very large or when we are dealing with the entire population rather than a finite sample). Likewise, the efficient estimation of the unknown parameters in econometrics is nothing but obtaining consistent estimators whose variances are as close to the Cramer-Rao variance bound as possible. Cramer-Rao bound and Rao-Blackwellization are important tools for this purpose. At a time when econometricians were analysing identification and estimation, model by model, Thomas Rothenberg (”Efficient Estimation with a Priori Information”, *Cowles Commission Monograph*, No. 23, 1973) took a more general approach. He showed that in a very broad class of models, local identification depends on the information matrix: if it is non- singular in a neighborhood of the true parameter value, the model is identified in that neighborhood. Thus he related identification to estimability concept of Rao. The problem of estimating the parameters of simultaneous equation systems in econometrics is shown by Rothenberg to be a special case of the general theory of statistical inference developed by Fisher and Rao, the basic tool being Cramer-Rao inequality. Rao-Blackwellization is very rarely used in econometrics and there is a lot of scope for further research in this area (For Rao-Blackwellization in econometrics see H. D. Vinod, "Rao-Blackwellization using Bootstraps for pre-test and instrumental variables estimator", *Journal of Quantitative Economics*, Vol. 20. 2022, Supplement, C. R. Rao Birth Centennial Issue.

Rao promoted applications of new statistical methods in econometrics by co-editing *Handbook of Statistics* on Econometrics with Maddala and Vinod (G. S. Maddala, C. R. Rao, and H. D. Vinod, *Handbook of Statistics*, Vol. 11, *Econometrics*, 1993, Amsterdam, North-Holland; G. S. Maddala, C. R. Rao, *Handbook of Statistics*, Vol. 14, *Finance*, 1996, Amsterdam, North-Holland; G. S. Maddala and C. R. Rao, *Handbook of Statistics*, Vol. 15, *Robust Inference*, 1997). For C. R. Rao’s contribution to the application of big data analysis in Finance see T. J. Rao ("Influence of C. R. Rao in financial statistics”, *Macroeconomics and Finance in Emerging Market Economies*, 2023).

In his *Econometric Theory* interview with Bera, Rao remarked that he was unhappy with economists not refining the measurements of variables, and in their not finding new relevant variables to improve prediction. Thus, he was suggesting the importance of errors in variables models and in devising methods to recognize the possibility of omitting relevant variables that are correlated with the included variables. It is easy to reformulate a model with omission of variables that are correlated with included variables as a model with included variables but with random coefficients (C. R. Rao, "The theory of Least Squares when the parameters are stochastic with application to the analysis of growth curves", *Biometrika*, Vol. 52, No. 3/4, 1965). Rao extended the standard linear models to linear random coefficient models. Thus, factor analysis models and models with random coefficients, and characterization of distributions, all developed by Rao, have a great future in economics. The method of instrumental variables plays an important role in obtaining consistent estimators. What is the best choice of an instrumental variable? This choice can benefit immensely from a differential geometric approach pioneered by Rao applied to the empirical likelihood function (Marriott, Paul, and Mark Salmon, *Application of differential geometry to econometrics*, Cambridge University Press, 2020).

For a more detailed description on Rao's work in econometrics one may see other references(G. S. Maddala, P. C. B. Phillip, and T. N. Srinivasan, *Advances in Econometrics and Quantitative Economics*, Wiley-Blackwell, 1995; Bera, Anil and Aman Ullah, "Rao's Score Test in Econometrics", *Journal of Quantitative Economics*, Vol. 7, No. 1: T. K. Kumar, H. D. Vinod, and S. Deman, "Dr. C. R. Rao's contributions to economic science", Proceedings of the Indian Academy of Sciences *Proceedings – Mathematical Sciences*, 130, 45 (2020); B. L. S. Prakasa Rao and H. D. Vinod (Editors), *Journal of Quantitative Economics*, (2022), Dr. C. R. Rao Birth Centenary Issue.

3.2 Rao's Contributions to other Social Sciences

Griliches showed that it was a mistake to abandon the errors in variables models just around the time that economic theory required their use. He formulated errors in variables model, with both observed and unobserved variables, as factor analysis models (Zwi Griliches, "Errors in variables and other unobservables" Henry Schultz Lecture, 1974, Econometric Society, *Econometrica*, Vol. 42, No. 6). This is a statistical model which was analysed from a multivariate statistics perspective by Rao and Slater twenty five years earlier in 1949 ("Multivariate analysis applied to differences between neurotic groups", *The British Journal of Mathematical and Statistical Psychology,* Vol. 2, Issue 1). Factor analysis models and the methods developed by Rao and Bartlett are now widely used in economics, sociology, psychology, and educational research. For an interface between economics, statistics, psychology, and sociology dealing with factor analysis one may see Grliches' paper cited above.

4 Rao's Influence on Survey Sampling Theory and Practice

One of the authors (Pathak) started his professional career with interest in survey sampling theory, literally and figuratively at the footsteps of C. R. Rao. His office was just opposite Rao's, and the typical routine used to be whenever Pathak had an exciting new result he would show it to Rao, and Rao would comment: "Interesting result but not of publishable significance". One of us, Kumar, used to see Rao go for tea and snacks in the afternoon mostly with two close friends S. Raja Rao and S. J. Poti. We also used to see D. B. Lahiri and M. N. Murthy of National Sample Survey Organization visit Rao at his office for consultations. Pramod recollects, that there were other instances when C. R. Rao would make significant revisions in his work before publication, e. g. Pramod's paper on the efficiency of Des Raj and Murthy's estimators in one such case. Salem H. Khamis, Des Raj, D. B. Lahiri, and M. N. Murthy who worked in close collaboration with Rao and significantly contributed to the theory and practice of sample surveys. As a student of Mahalanobis, Rao's initial work on survey sampling was with the anthropometric survey in 1941 in which he developed the random permutation model for sampling (T.

J. Rao, "C. R. Rao's Influence on theory and practice of sample surveys", *Journal of Indian Society of Probability and Statistics*, 2020). Rao was very much involved with the National Sample Survey Organization in the sampling designs, setting up sample survey questionnaires, and in their analyses. It was V. P. Godambe's paper ("A Unified Theory of Sampling From Finite Populations", *Journal of the Royal Statistical Society. Series B* (Methodological), Vol. 17, No. 2 (1955), and his visiting position as a Senior Research Fellow at I. S. I. during 1958-1959 that stimulated the interest of Rao, Basu, and Pathak to apply the theory of statistical inference to sample survey methods. It was S. Raja Rao of the National Sample Survey Organization in Kolkata, a close friend of C. R. Rao, who brought Rao's attention to the importance of the concept of sufficiency in sample surveys. This concept of sufficiency and the associated Rao-Blackwellization were followed up by Basu and Pathak (D. Basu, "On sampling with and without replacement", *Sankhya*, Vol. 20, 1958, and P. K. Pathak, "On simple random sampling with replacement", *Sankhya*, Vol. 24, Ser A, 1962, and P. K. Pathak, "Sufficiency in sampling theory", *Annals of Mathematical Statistics*, Vol. 35, 1964). Hartley and Rao noted that the non-existence of minimum variance unbiased estimator was due to the class of estimators considered by Godambe, that depended on identification labels. They showed that once that dependence is dropped appropriately, Minimum Variance Unbiased Estimator (MVUE) exists (H. O. Hartley and J. N. K. Rao "A new estimation theory for sample surveys", *Biometrika*, Vol 58, No. 3, 1968).

Some of these developments resulted in pointing out serious problems when we make inferences with survey sampling with finite populations, problems such as the non-existence of MVUE, the likelihood surface being flat in the direction of some parameters, whether one should look for Rao-Blackwellization with a bad sample or a bad sample design or look for a better samples or better sampling designs. The non-existence of MVUE was also addressed by advancing various admissibility criteria without any practical significance. This prompted Rao in 1968 to request J. N. K. Rao, who was then visiting I. S. I., to deliver a few lectures on inference issues in sample surveys. After attending those lectures Rao prepared two papers addressing these issues (C. R. Rao, "Some aspects of statistical inference in problems of sampling from finite populations, (with discussion and reply by author)", In: V. P. Godambe and D. A. Sprott (eds.) *Foundations of Statistical Inference*, 1971, Holt, Rinehart and Winston of Canada; and Rao, C. R. "Some Problems of Sample Surveys", *Advances in Applied Probability*, Vol. 7, Sept. 1975). Rao reformulated the issues using a basic probabilistic structure to the inference problems along the lines of Kolmogrov's measure theoretic formulation. Noting that the question posed and the answer obtained in statistical inference depend on the sample space, parameter space, and the probability distribution assumed for the generation of the observations, Rao concluded: However, many controversies can be avoided if the basic issues are clearly stated and statisticians do not insist on a monolithic structure for all problems of statistical inference. Much damage has been done by fashions and slogans in statistics introduced by theoretical statisticians who have no experience of handling live data and extracting information from them. Jointly with Mitra, Mathai and Ramamurthy, Rao edited the *Tables for Statistical Work* (C. R. Rao, S. K. Mitra, A. Mathai and K. G. Ramamurthy (1975), *Formulae and Tables for Statistical Work*, 1966, Statistical Publishing Society, Calcutta). This book included a brief review of basic sample survey estimates and their standard errors, in addition to random sample numbers and random permutations. These chapters of the tables are often consulted by students, scholars and survey practitioners. Here we also suggest referring to two volumes of the *Handbook of Statistics* that Rao edited one with P. R. Krishnaiah in 1988 (*Sampling*, Volume 6) and the second with D. Pfeffermann in 2009 (*Sample surveys: Design, Methods and Applications*, Volume 29A).

5 Conclusions

In conclusion, we believe, the teaching and research tradition set by the legend C. R. Rao continues worldwide and his students, textbook users, collaborators, and friends take forward the rich culture of theoretical and applied research. The C. R. Rao Advanced Institute in Hyderabad, India started to conduct high-class original work in all the research areas that C. R. Rao inspired. We wish good luck to all the scientists. Finally, it gives us a great feeling to know the legend personally who lived a cheerful and inspiring life till the end.

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